P425/1
PURE MATHEMATICS
Paper 1
Sept. 2022
3 hours

VECTORS TEST

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and only **five** in section **B**.*

*Any additional question(s) answered will **not** be marked.*

*Each question in section **A** carries **5** marks while each question in section **B** carries **12** marks.*

*All working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Silent , non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Show that the points A(1,2,3), B(3,8,1) and C(7,20,-3) are collinear.

   (05 marks)

2. OAB is a triangle. X and Y are the midpoints of **OA** and **AB** respectively. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are position vectors of points A and B respectively. Find in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ the position vector of Z, the point of intersection of **XB** and **OY**.

   (05 marks)

3. Find the angle between the y-axis and the line $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{k} + \lambda(4\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k})$. State the direction ratios of the given line. *(05 marks)*

4. Show that the lines $\boldsymbol{r_1} = \begin{pmatrix} 17 \\ 2 \\ -6 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -3 \\ 9 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} 6 \\ 7 \\ -1 \end{pmatrix}$ are skew. *(05 marks)*

5. Find the equation of plane in scalar product form that contains the points P(1,1,1), Q(1,-2,-1) and R(3,-1,2). *(05 marks)*

6. Find the foot of the perpendicular from the point A(25,5,7) to the plane with equation $12x + 4y + 3z = 3$. *(05 marks)*

7. Show that the points M(2,21) and N(1,-2,1) lie on the opposite sides of the plane with equation $2x - 3y + z = 1$. *(05 marks)*

8. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-1}{1}, z = 1$. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt only **five** questions from this section.*

9. (a) Show that the points A,B and C with position vectors $3\boldsymbol{i} - 4\boldsymbol{j}, 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $7\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ are vertices of a triangle. Hence find the area of the triangle. *(07 marks)*

(b) PQRS is a quadrilateral with P(2,2), Q(5,-1),R(6,-2) and R(3,1). Identify the type of quadrilateral using vector methods. *(05 marks)*

10.(a) Find the coordinates of the point of intersection of the plane $2x - y + 4z = 32$ and the line $\frac{1-x}{-3} = \frac{y+2}{2} = \frac{z-1}{5}$. Obtain the angle between the line and the plane. *(08 marks)*

(b) Find whether the points A(2,-2,5) and B(1,2-1) lie on the plane in (a) above. *(04 marks)*

11.The equations of the line L and the plane π are $\frac{x-2}{1} = \frac{2-y}{-2} = \frac{z-3}{3}$ and $2x + y + 4z = 13$. P is a point on the line where $x = 3$. N is the foot of the perpendicular from point P to the plane. Find

(a) The coordinates of the point N. *(08 marks)*

(b) Equation of the plane through (1,2,-1) perpendicular to line **NP**.

(04 marks)

12.The vector equations of 2 lines are $\boldsymbol{r_1} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} 2 \\ 2 \\ t \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ where t is a constant. If the two lines intersect at C, find

(a) The value of t and the coordinates of C *(07 marks)*

(b) Find the cartesian equation of the plane containing the two lines.

(05 marks)

13.(a) The points A and B has position vectors a and b respectively with respect to the origin O. Point C divides line **AB** externally in the ratio λ: μ. Show that $\boldsymbol{OC} = \frac{\lambda \boldsymbol{b} - \mu \boldsymbol{a}}{\lambda - \mu}$. *(04 marks)*

(b) The coordinates of points A and B are (-3, -1, -2) and (5,3,6) respectively. If point P divides **AB** externally in the ratio 2:3 and point Q divides **AB** externally in the ratio 5:1, find the coordinates of R the midpoint of **PQ**. *(05 marks)*

(c) Find the equation of line through point R in (b) above and perpendicular to the plane $2x - 3y + z = 5$. *(03 marks)*

14. A triangle OPR is such that $\boldsymbol{OR} = \boldsymbol{r}$ and $\boldsymbol{OP} = \boldsymbol{p}$**.** S and Q are points produced on **OR** and **OP** such that $\boldsymbol{OS} = 2\boldsymbol{r}$ and $2\boldsymbol{OQ} = 3\boldsymbol{p}$ respectively. K is the point of intersection of **PS** and **QR.** Show that K divides **PS** in the ratio 1:3. *(12 marks)*

15. (a) Find the equation of plane that contains the points A(1,2,3) and B(2,-1,2) and parallel to the line $\frac{x-2}{1} = \frac{2-y}{-2} = \frac{z-3}{3}$. *(05 marks)*

(b) A right circular cone has its vertex at the point (2,1,3) and the centre of its plane face at the point (1,-1,2). The generator of the cone (slanting side) has equation $\boldsymbol{r} = (2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}) + t(\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$. Find the radius of the cone. *(07 marks)*

16. (a) Show that the shortest distance from the plane with equation $ax + by + cz = d$ to the point $A(p, q, r)$ is given by $\frac{|ap+bq+cr-d|}{\sqrt{a^2+b^2+c^2}}$.

(05 marks)

(b) Obtain the distance from the origin to the plane $2x - 6y + 3z = 14$.

(03 marks)

(c) Calculate the distance between the planes $2x - 6y + 3z = 14$ and $4x - 12y + 6z + 21 = 0$. *(04 marks)*

GOOD LUCK